Some notions of largeness relative to filters and their algebraic characterizations

Given two filters $\mathcal{F}$ and $\mathcal{G}$, Shuungula, Zelenyuk, and Zelenyuk defined the concept of a $(\mathcal{F}, \mathcal{G})$-syndetic subset of a discrete semigroup, using this notion to characterize the closure of the smallest ideal in closed subsemigroups of $\beta S$. We define a “filtered” notion of a thick subset, generalizing the usual combinatorial notion of thick, and establish its duality with the filtered notion of syndeticity. Exploiting this duality yields combinatorial and algebraic characterizations that generalize similar characterizations of closed subsemigroups, closed left ideals, and closed right ideals in $\beta S$, extending results originally proved by Davenport.