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Monochromatic configurations in finite colorings of $\mathbb{N}$

Is it possible to color the natural numbers with finitely many colors, so that whenever x and y are of the same color, their sum $x+y$ has a different color? A 1916 theorem of I. Schur tells us that the answer is no. In other words, for any finite coloring of $\mathbb{N}$, there exist x and y such that the triple $\{x, y, x+y\}$ is monochromatic (i.e. has all terms have the same color). A similar result holds if one replaces the sum $x+y$ with the product xy, however, it is still unknown whether one can finitely color the natural numbers in a way that no quadruple $\{x, y, x+y, xy\}$ is monochromatic! In this talk I present a recent partial solution to this problem, showing that any finite coloring of the natural numbers yields a monochromatic triple $\{x, x+y, xy\}$.